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Helioseismology and p+p $\rightarrow$ d + e^+ + ν_e in the sun

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Abstract

By using a phenomenological field theory of nucleon-nucleon interactions, Oberhummer et al. found a cross section of p+p $\rightarrow$ d + e^+ + ν_e about 2.9 times that given by the potential approach and adopted in Standard Solar Model calculations. We show that a solar model with $S = 2.9 S_{SSM}$ is inconsistent with helioseismic data, the difference between model predictions and helioseismic determinations being typically a factor ten larger than estimated uncertainties. We also show that, according to helioseismology, S cannot differ from S_{SSM} by more than 15%.

The rate of the initial reaction in the pp chain is too low to be directly measured in the laboratory (even in the solar center this rate is extremely small, of the order of 10^{-10} yr^{-1} consistently with the solar age) and it can be determined only by using the theory of low energy weak interactions, together with the measured properties of the deuteron and of the proton-proton scattering. In terms of the astrophysical factor, S(E), what really matters is its zero energy value, which for brevity will be indicated simply as S. While we refer to [1–3] for updated reviews, we remark that the calculated values are all in the range (3.89–4.21) 10^{-25} MeV b, i.e. they differ from their mean by no more than 3%. In summary, as input of Standard Solar Model (SSM) calculations, one takes [2]:

$$S_{SSM} = 3.89 \cdot 10^{-25}(1 \pm 0.01) \ \text{MeV b} \quad . \tag{1}$$

Although some warning is in order as to the meaning of the quoted error, one may conclude that well known physics determines S to the level of few per cent or even better.

Recently, however, Oberhummer et al. [4] presented a new evaluation of S in a relativistic field theory framework, where strong interactions are phenomenologically described by one nucleon loop diagrams. As well known since [5], Adler-Bell-Jackiw anomalies are present in such models. The authors of Ref. [4] claim that such anomalies provide the dominant contributions to the scattering amplitude, and that this effect has not be considered in

the potential approach, yielding to Eq. (1), as being the result of a purely field theory phenomenon without a classical analogue. The estimated reaction rate is a factor 2.9 times that of the conventional approach.

Although it is not clear to us if the proposed field theory is suitable for an accurate description of the low energy strong interaction phenomenology (deuteron wave function, nucleon-nucleon scattering amplitudes,...), it is of some interest to reconsider the effect of varying S well beyond its estimated uncertainty, see Eq.(1).

The effect on the central solar temperature and on neutrino fluxes has been discussed e.g. in [6–9].

As well known, and remarked in [4], a drastic increase of S alleviates but does not solve the solar neutrino puzzle. The resulting low central temperature model implies a drastic reduction of ^{7}Be neutrinos and an even stronger one for ^{8}B neutrinos. As a consequence, the predicted Gallium signal is close to the observed values, but then Homestake, Kamiokande and Superkamiokande are observing definitely too many (^{7}Be and/or ^{8}B) neutrinos!

In this letter we shall concentrate on helioseismic implications of varying S.

Indeed, helioseismology allows us to look into the deep interior of the Sun, probably more efficiently than neutrinos (for reviews see [10–14]). The highly precise measurements of frequencies and the tremendous number of measured lines enable us to extract the values of sound speed and density inside the Sun with accuracy better than 1%. Furthermore, from helioseismic data one derives accurate predictions on some properties of the convective envelope: the transition of the temperature gradient between being subadiabatic and adiabatic at the base of the solar convective zone gives rise to a clear signature in the sound speed [13]. Helioseismic measurements therefore determine the location R_b and the density ρ_b of the base of the convective zone. In addition, the photospheric helium abundance Y_{ph}, which is of fundamental importance both to cosmology and to solar structure theory and which cannot be determined by direct measurements, is constrained by helioseismology. In Table I we present the helioseismic determination of the above mentioned quantities R_b, ρ_b and Y_{ph}, together with *conservative* estimates of the uncertainties due to both observational errors and inversion technique, see Ref. [15].

Recent standard solar model calculations, including element diffusion and using updated opacities and accurate equations of state, are well in agreement with helioseismic data, see Ref. [15]. Let us compare with helioseismic data a solar model (hereafter MOD2.9) obtained from the FRANEC evolutionary code [16] by taking $S = 2.9 S_{SSM}$, all other input parameters being kept at the SSM values.

Concerning the (isothermal) sound speed squared, $U = P/\rho$, the estimated accuracy of helioseismic determination is, conservatively, of order 0.5% for intermediate values of the solar radial coordinate R. More precisely, as a function of $R/R_\odot$, the relative accuracy of U corresponds to the dotted area in Fig. 1. From the same figure one sees that the SSM satisfies the helioseismic constraint almost everywhere, in that the error band generally includes $(U_{SSM} - U_\odot)/U_\odot$ where U_{SSM} is the value predicted by the SSM and $U_\odot$ is the helioseismic determination.

On the other hand, for MOD2.9 the profile of $(U_{2.9} - U_\odot)/U_\odot$ looks clearly inconsistent with helioseismology. At $R \simeq 0.6 R_\odot$ the relative difference is of order 5%, a factor ten

beyond the estimated uncertainty of $U_\odot$ [1].

The comparison between the properties of the convective envelope, see Table I and Fig. 2, also shows the inadequacy of MOD2.9. For instance, the distance between the predicted and the true depth of the convective zone is ten times the estimated error.

All this shows that $S = 2.9 S_{SSM}$ is untenable. On the other hand, we remind that only theoretical estimates of S are available and observational information would be welcome. In this respect, it is interesting to determine the range of S-values which are acceptable in comparison with helioseismology.

We remind that there are two major uncertainties in building SSMs: solar opacity κ and heavy element abundance ζ =Z/X are only known with an accuracy of about $\pm$ 10%. By using κ and ζ as free parameter within their estimated uncertainties we can determine the acceptable range of S as that such that R_b, ρ_b and Y_{ph} are all predicted within the helioseismic range.

The dependence of these quantities on κ, ζ and S has been determined numerically in Ref. [19]:

$$R_b = R_{b,SSM} \left(\frac{\kappa}{\kappa_{SSM}}\right)^{-0.0084} \left(\frac{\zeta}{\zeta_{SSM}}\right)^{-0.046} \left(\frac{S}{S_{SSM}}\right)^{-0.058} \tag{2a}$$

$$\rho_b = \rho_{b,SSM} \left(\frac{\kappa}{\kappa_{SSM}}\right)^{0.095} \left(\frac{\zeta}{\zeta_{SSM}}\right)^{0.47} \left(\frac{S}{S_{SSM}}\right)^{0.86} \tag{2b}$$

$$Y_{ph} = Y_{ph,SSM} \left(\frac{\kappa}{\kappa_{SSM}}\right)^{0.61} \left(\frac{\zeta}{\zeta_{SSM}}\right)^{0.31} \left(\frac{S}{S_{SSM}}\right)^{0.14} \tag{2c}$$

Most of the information on S arises from data on ρ_b as this observable depends strongly on S whereas it is weakly affected by the others parameters. One can understand the dependence on S, at least qualitatively. A value of S larger than S_{SSM} implies smaller temperature in the solar interior, which thus becomes more opaque (in other words, the region of partial ionization is deeper). Radiative transport therefore is less efficient and convection starts deeper in the Sun ($R_b < R_{b,SSM}$) where density is higher ($\rho_b > \rho_{b,SSM}$).

By using Eqs. (2) and the allowed ranges reported in Table I, also taking into account the predictions of different SSMs, we find:

$$0.94 \leq S/S_{SSM} \leq 1.18 \tag{3}$$

In conclusion, we remark that helioseismology provides the only observational constraint, although indirect, on the p+p $\rightarrow$ d + e^+ + ν_e reaction.

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[1] We remark that sound speed profiles for $S \neq S_{SSM}$ have been discussed in Refs. [17,18]

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TABLES

TABLE I. For the depth of the convective zone R_b, the density at the bottom of the convective zone ρ_b and the photospheric helium abundance Y_{ph} we present the helioseismic determination, from Ref. [15], and the predictions corresponding to $S = 2.9 S_{SSM}$.

Q	Helioseismology	MOD2.9
$R_b/R_\odot$	0.711 $\pm$0.003	0.677
ρ_b [g/cm^3]	0.192 $\pm$0.007	0.409
Y_{ph}	0.249 $\pm$0.011	0.270

FIGURES

FIG. 1. The fractional difference $(U - U_\odot)/U_\odot$ for the FRANEC-SSM (solid line) and for the model with $S = 2.9 S_{SSM}$ (dashed line). The dotted area corresponds to the uncertainty on $U_\odot$.

FIG. 2. For the indicated quantities Q we present the fractional difference $(Q - Q_\odot)/Q_\odot$ for $S = 2.9 S_{SSM}$ (diamonds) together with the relative helioseismic uncertainties (bars).

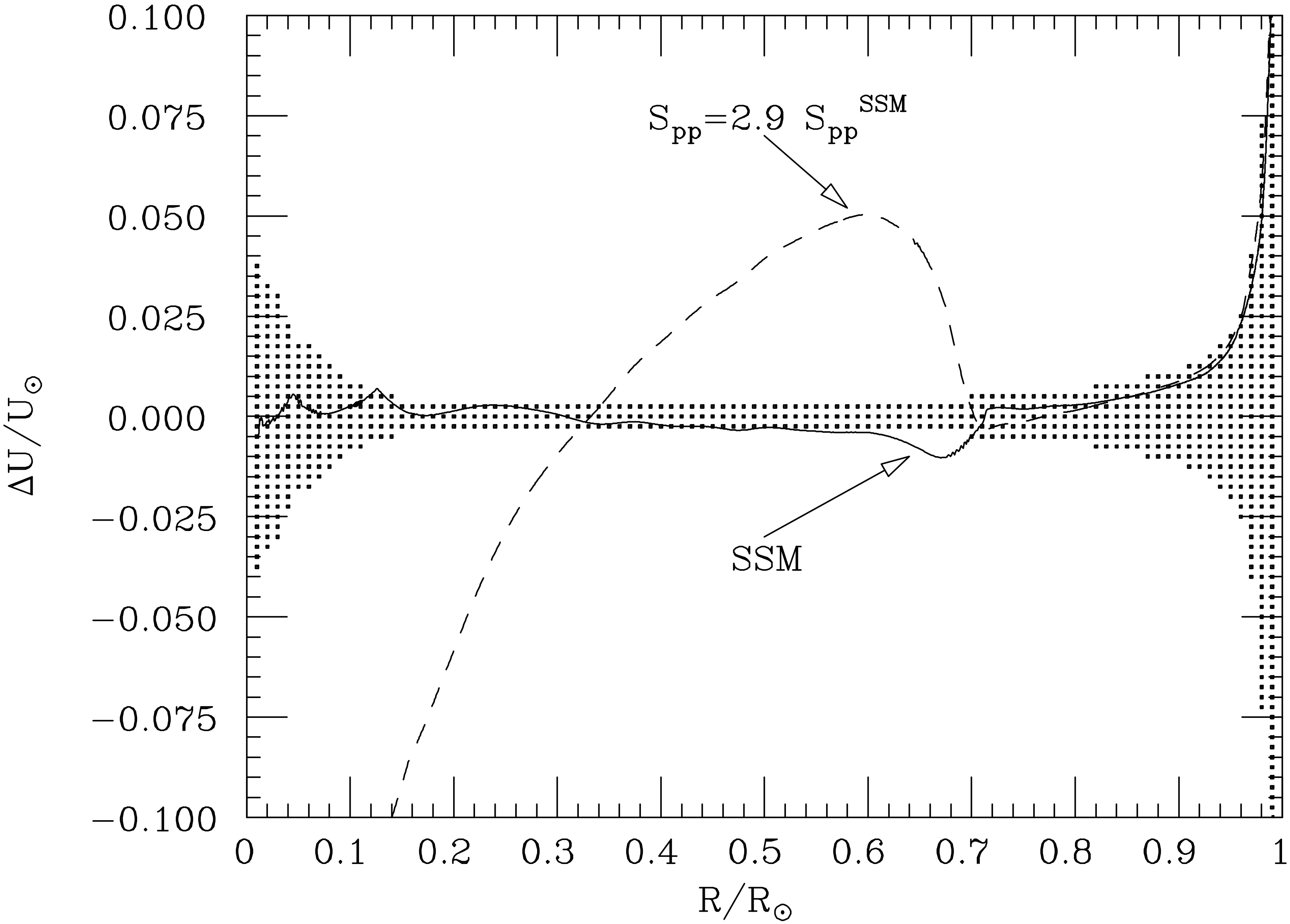
0.100
0.075
0.050
0.025
0.000
−0.025
−0.050
−0.075
−0.100
$\Delta U/U_\odot$
S_{pp}=2.9 S_{pp}^{SSM}
SSM
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
$R/R_\odot$

Fig. 1

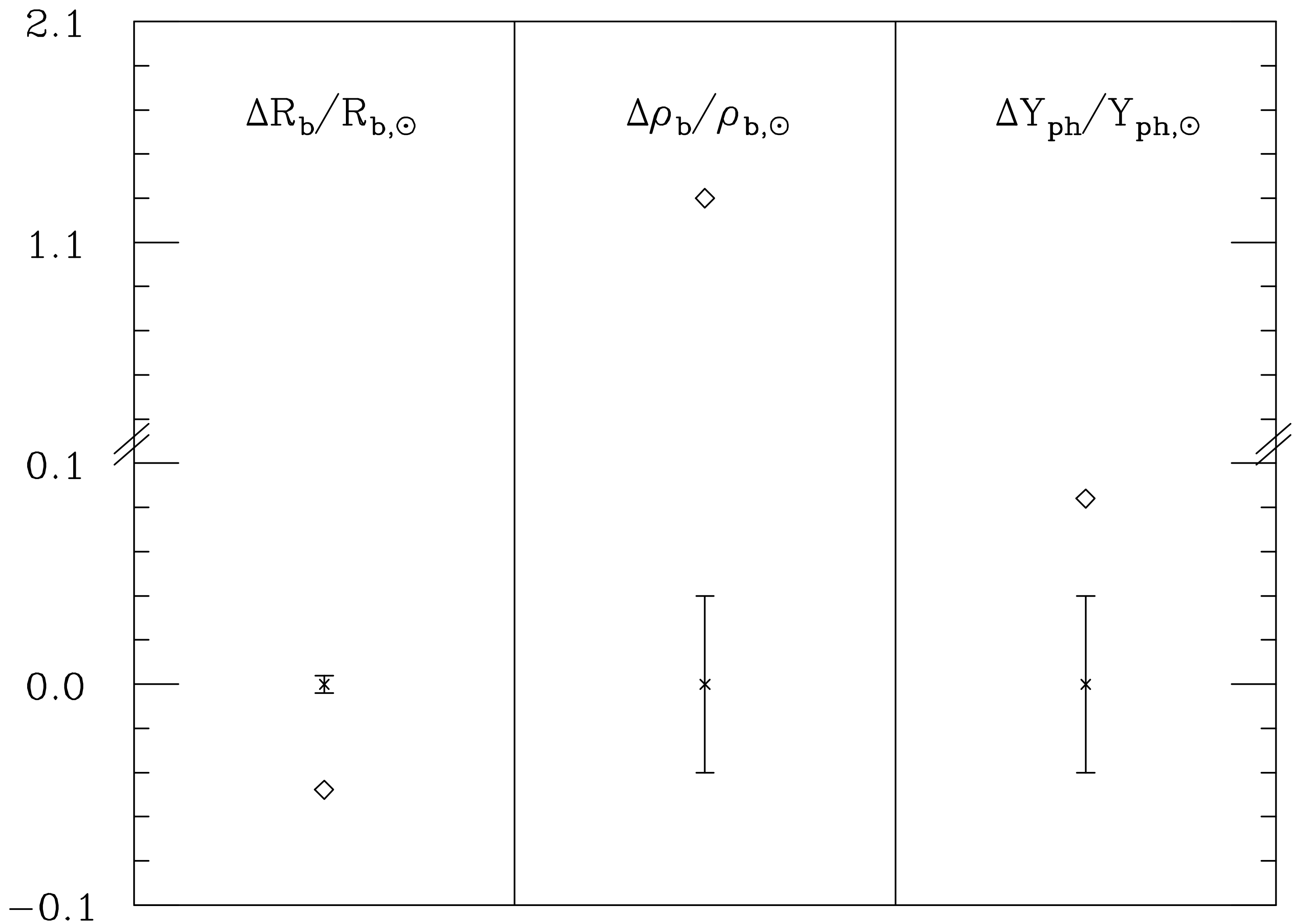
2.1
1.1
0.1
0.0
−0.1
$\Delta R_b/R_{b,\odot}$
$\Delta\rho_b/\rho_{b,\odot}$
$\Delta Y_{ph}/Y_{ph,\odot}$

Fig. 2